MJC I Semester I # 4.1 Unit-04

# #4.1 Matrices, operation on matrices, kinds of matrices

## #4.1.1 Definition of Matrix :-

A matrix is a rectangular array of numbers written in the form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}]_{m \times n}$$

$m$ → Row, $n$ → Column

where $a_{ij}$ are real or complex numbers.

(i) Matrices are generally represented by [ ] or ( ) or || ||

(ii) Matrix is denoted by Capital letter of English alphabet.

(iii) $a_{ij}$ is the element of matrix.

(iv) The order of matrix is $m \times n$ (read as m by n)

Ex (1)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}_{3\times 4}$$

(i) No of rows = 3

(ii) No of columns = 4

(iii) order of matrix = $3\times 4$ (3 by 4)

(iv) $a_{11} = 1$, $a_{22} = 6$, $a_{23} = 7$, $a_{34} = 12$ etc

(v) No of elements = $3 \times 4 = 12$

## #4.1.2 Kinds of Matrices :-

(1) Row matrix :- A matrix having only one row is called row matrix.

$A = [a_{ij}]_{m\times n}$ is called row matrix if $m = 1$

Ex (1) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1\times 4}$

(ii) Column Matrix :- A matrix having only one col( is called column matrix.

i.e. $A = [a_{ij}]_{m \times n}$ is called column matrix if $n = 1$.

Ex ① $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times 1}$

(iii) Square matrix :- A matrix in which the number of rows is equal to the number of col is called square matrix.

$A = [a_{ij}]_{m \times n}$ is called square matrix if $m = n$.

(iv) Diagonal matrix :-

$A = [a_{ij}]_{m \times n}$ is called diagonal matrix

if ① $m = n$

(ii) $a_{ij} = 0, \ i \neq j$

(iii) $a_{ij} \neq 0, \ i = j$

(v) Scalar matrix :-

$A = [a_{ij}]_{m \times n}$ is called scalar matrix

if ① $m = n$

(ii) $a_{ij} = 0, \ i \neq j$

(iii) $a_{ij} = k, \ i = j$

(vi) Unit or Identity matrix :-

$A = [a_{ij}]_{m \times n}$ is called unit or Identity matrix

if ① $m = n$

(ii) $a_{ij} = 0 \quad i \neq j$

(iii) $a_{ij} = 1 \quad i = j$

It is denoted by by $I_n$ or $I$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$

(vii) Zero or Null matrix :- A matrix having all elements are zero, called zero or null matrix. It is denoted by $O_{m\times n}$

$$\therefore O_{2\times2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad O_{2\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3}$$

(viii) Upper Triangular matrix :- A Square matrix $A = [a_{ij}]_{n\times n}$ is called upper triangular matrix if $i > j$

(ix) Lower Triangular matrix :- A Square matrix $A = [a_{ij}]_{n\times n}$ is called lower triangular matrix if $i < j$.

(X) Commutative and Anti-Commutative Matrices :-

(a) If $AB = BA$ then A and B are called Commutative matrix

(b) If $AB \neq BA$ then A and B are called Anti Commutative matrix

(xi) Periodic Matrices :-

(a) Idempotent matrix :- If $A^2 = A$ then A is called indempotent matrix.

(b) Involuntary matrix :- If $A^2 = I$ then A is called involuntary matrix

(c) Nilpotent matrix :- If $A^{k+1} = 0$, but $A^k \neq 0$ then A is called nilpotent.

# Equality of two matrices :-

Two matrices A and B are of same types are called Equal if $a_{ij} = b_{ij}$

$A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$ Then $A = B$

$\Rightarrow a_{ij} = b_{ij}$

Scanned with OKEN Scanner

# Symmetric and Skew Symmetric matrix :-

① A Square matrix is called Symmetric if $A' = A$

or, $[a_{ij}]_{n \times n}$ is called Symmetric if $a_{ij} = a_{ji}$

• Properties of Symmetric matrix :-

① Addition and difference of two Symmetric matrices is also Symmetric

(ii) If A and B are two Symmetric matrices. Then $AB = BA$ if and only if $AB$ is a symmetric matrix.

(iii) If A is Symmetric then $A^n$ is also Symmetric.

(iv) If A is Symmetric then $A^{-1}$ is also Symmetric.

② Skew-Symmetric Matrix :- A Square matrix A is called Skew Symmetric if $A' = -A$

or, A Square matrix $A = [a_{ij}]_{n \times n}$ is called Skew-Symmetric if $a_{ij} = -a_{ji}$

Note: ① The diagonal elements of a Skew Symmetric matrix are zero.

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

• ~~Properties of Skew Symmetric~~

(i) The Sum of two Skew Symmetric is also Skew Symmetric.

(ii) Every Square matrix can be expressed uniquely as the Sum of Symmetric and Skew Symmetric matrices.

# Properties of Symmetric and Skew Symmetric :-

Theorem ① For any Square matrix A with real number elements ① $A + A'$ is a Symmetric matrix.

(ii) $A - A'$ is a Skew Symmetric matrix.

Proof :-

① $(A + A')' = A' + (A')'$ {Properties of transpose}

$= A' + A = (A + A')$

$\therefore A + A'$ is symmetric

(ii) $(A - A')' = A' - (A')'$

$= A' - A = -(A - A')$

$\therefore A - A'$ is skew Symmetric